

Muon Cooling in a Quadrupole Magnet Channel

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Abstract

As discussed before,[1] a cooling channel using quadrupole magnets in a FODO transport channel can be used for initial cooling of muons. In the present note we discuss this possibility of a FODO focusing channel for cooling, and we present ICOOL simulations of muon cooling within a FODO channel. We explore a 1.5m cell-length cooling channel that could be used for the initial transverse cooling stage of a muon collider or neutrino factory.

Introduction

Many of our initial studies have used solenoidal focusing for the capture of low-energy pions from a target, and cooling of the muons, primarily because solenoids focus in both x and y and are reasonably effective in obtaining small β_{\perp} . However initial cooling does not need very small β_{\perp} . C. Johnstone et al. have suggested that initial muon capture and ionization cooling could occur within a short-period FODO quad lattice, and calculated an example in COSY.[1]

The study 2A [2] cooling and transport had a β_{\perp} of $\sim 0.8\text{m}$, obtained from 0.75m long solenoidal cells. We initially consider a 0.75m (half-cell) FODO quad channel as an alternative. This lattice has a β_x and β_y that both oscillate between $\sim 2.5\text{m}$ and $\sim 0.9\text{m}$, with an average of ~ 1.75 . The lattice does not focus simultaneously in x and y, so only an average value of β_{\perp} can be used for cooling. The β_{\perp} is sufficiently small for initial cooling, particularly if the material is a low-scattering material such as H_2 .

Muon cooling within a beam transport can be estimated using the rms cooling equation.[3] The cooling method used is ionization cooling, which is limited by the beam focusing and multiple scattering in the cooling absorbers. The basic cooling equation for transverse muon cooling within energy loss absorbers is:

$$\frac{d\epsilon_N}{ds} = -\frac{g_t}{P_{\mu}} \frac{dP_{\mu}}{ds} \epsilon_N + \frac{\beta_{\perp} E_s^2}{2\beta^2 m_{\mu} c^3 L_R P_{\mu}}, \quad [1]$$

where ϵ_N is the transverse normalized emittance, g_t is the transverse partition number ($g_t=1$ without emittance exchange), $\beta = v/c$, P_{μ} is the muon momentum, dP_{μ}/ds is the momentum loss in the absorber, E_s is the characteristic scattering energy ($\sim 14\text{ MeV}$), L_R is the characteristic radiation length in the material, and β_{\perp} is the transverse focusing function at the absorber. Cooling is limited to the equilibrium transverse emittance, which is:

$$\epsilon_{N,eq} = \frac{\beta_{\perp} E_s^2}{2g_t \beta m_{\mu} c^2 L_R \frac{dE}{ds}}, \quad [2]$$

where $dE/ds = \beta c dP_{\mu}/ds$. dE/ds is approximately given by the Bethe-Bloch formula :

$$\frac{dE}{ds} \cong 4\pi N_A \rho r_e^2 m_e c^2 \frac{Z}{A} \left[\frac{1}{\beta^2} \ln \left(\frac{2m_e c^2 \gamma^2 \beta^2}{I(Z)} \right) - 1 - \frac{\delta}{2\beta^2} \right] \quad [3]$$

where N_A is Avogadro's number, Z , A are the material atomic number and weight, ρ is the material density and $\delta \approx 0$ is the density effect parameter ($\delta = \sim 0$). The material with largest $L_R dE/ds$ is hydrogen, and therefore the best cooling material. $I(Z)$ is the material ionization energy, and is approximately $16 Z^{0.9}$ eV.

The parameters in equation [2] are functions of the cooling material, and the focusing parameters β_{\perp} , and the beam speed β , with $\beta \cong 1$ for For a hydrogen absorber the value (at $g_t = 1$) is $\epsilon_{N,eq} = \sim 0.0037 \beta_{\perp}$ and for LiH it is $\sim 0.0068 \beta_{\perp}$.

In the Neutrino factory study, a solenoidal focusing transport with a period of 0.75m was used for initial cooling of the muons. For initial focusing a relatively weak focusing was used; the average β_{\perp} was ~ 0.8 m, and the Study 2A study had ~ 100 m of such cells.

Solenoidal focusing imposes helical motion upon the particle orbits. This additional motion places a momentum-amplitude correlation for synchronous motion, complicating matching of the beam within rf accelerating systems. Solenoidal channels have losses due to mismatch of the momentum-amplitude correlation, and additional resonance from the longitudinal/transverse coupling.

Quadrupole focusing has transverse and longitudinal motion more clearly separated than with solenoids, and amplitude-momentum correlations do not limit the acceptance. Using quadrupoles for focusing requires alternating quad signs along the transport. A simplest configuration for a focusing quad transport is the FODO lattice, which consists of a sequence of cells with an F quad, an O drift, an opposite sign D quad and another drift. A FODO transport has a relatively large momentum acceptance; all particles with momentum greater than a small-momentum can be stably transported.

The disadvantage of the FODO transport is that it does not have a focus point with small β -function in both x and y. There is therefore no preferred absorber location with minimal emittance growth from scattering in both x and y. However that also means that the absorber could be diffused throughout the transport without a cooling penalty, as could be done with gaseous hydrogen.

In a quadrupole focusing system with bending magnets, a horizontal dispersion is generated. That dispersion, with wedge absorbers, can be used to obtain emittance exchange cooling, coupling the x-motion and the longitudinal motion, with the y-motion and cooling unchanged. Dispersion with wedges can also be used for emittance exchange

cooling within solenoids, but the x and y motions are so tightly coupled that separating x and y exchange is not practical. (The coupling may or may not be desirable.)

Sample parameters

As an initial example, we consider a modified version of the lattice used in the study 2A cooling channel. That lattice was an alternating solenoid lattice with a period of 0.75m. The 0.75m lattice included 0.5m of rf cavity plus 2 1cm LiH absorbers, with the remaining space allowing room for focusing solenoidal coils. The betatron function within that lattice was $\sim 0.8\text{m}$, with only a weak variation along the cell length.

For the FODO lattice we choose a half-cell period of 0.75m with a quad length of 0.23m; the remaining 0.52m is reserved for absorbers and RF. Each half-cell contains two 200.7 MHz pillbox rf cavities with 0.25m length operating at 15.25 MV/m gradient. The reference momentum for the neutrino factory cooling channel is 215MeV/c. If we set the phase advance per cell at 60° , the required gradient is 4.15T/m, which implies a field of 1.66T at a reference radius of 0.4m. This field could be reached with normal-conducting ferric magnets as well as superconducting magnets. The maximum betatron function is $\sim 2.6\text{m}$ and the minimum is $\sim 0.9\text{m}$, with an average value of $\sim 1.6\text{m}$. This is significantly larger than in the solenoidal channels.

The μ beam emittance from the target and capture transport is $\sim 0.02\text{m}$ (rms, normalized). For simulation studies we initially placed a beam with these rms emittances at the start of the cooling transport. We consider two cases: one in which the absorbers were 1cm LiH absorbers, placed as in the Study 2A report (2 per 0.75m half-cell), and the second, in which the absorber is gaseous hydrogen which fills the beam transport at a density corresponding to 133 atm at 293°K. The average energy rate loss is the same for both cases ($\sim 3.5\text{MeV} / 0.75\text{m}$ transport).

Simulations of muon cooling were run using the ICOOL code,[4] with a 97.5m long cooling channel (65 cells). For the LiH absorbers, the transverse emittance $\epsilon_{t,N}$ cools from $\sim 0.0204\text{m}$ to 0.015m in $\sim 25\text{m}$, and cools to $\sim 0.012\text{m}$ in $\sim 50\text{m}$. The asymptotic cooled emittance is $\sim 0.010\text{m}$. These results were obtained using the newer ICOOL scattering model based on Fano (ICOOL model 6).[5] With the older Bethe model (ICOOL model 4) the initial cooling rate is the same but the asymptotic value is $\sim 0.0115\text{m}$, which is somewhat larger.

With hydrogen gas-filled transport, the transverse emittance $\epsilon_{t,N}$ cools from $\sim 0.0204\text{m}$ to 0.015m in $\sim 20\text{m}$, and cools to $\sim 0.010\text{m}$ in $\sim 50\text{m}$, and is cooled further to $\sim 0.0065\text{m}$ at $\sim 100\text{m}$. The asymptotic cooled emittance is $\sim 0.005\text{m}$. The difference between scattering models 4 and 6 was small, much less than in the LiH simulations. The hydrogen-filled gas absorber had performance similar to the Study 2A cooling channel solenoid case with LiH absorbers, indicating that the quad channel could be used as a substitute for that cooling channel, with similar potential performance. The cooling with gas absorbers is somewhat better than the simple rms formula; the equilibrium emittance is $\sim 30\%$ less

than a simple insertion of the values would indicate. Figure 2 shows these changes in emittance along the cooling channel for the cases of LiH absorbers and H₂ gas absorbers.

Unlike solenoidal channels there is no large amplitude-energy correlation, and a much reduced problem of beam loss in matching and transport through the channel because of that correlation.

A significant advantage of the quad channel is that the magnetic field in the rf cavities is much weaker and is transverse rather than parallel to the accelerating field. Initial MTA observations indicate that solenoidal fields may limit the peak gradient achievable in pillbox cavities, and it is likely that the weaker transverse fields may not limit the gradients.[6] Initial experiments with pillbox cavities within magnetic fields indicate a large reduction in possible gradient because of this effect.

Other Examples

Our baseline example corresponded to a phase advance of 60°/ cell at 215 MeV/c. We also considered a case with 90°/ cell, obtained by increasing the quad gradient to 5.88 T/m. This focuses the beam to a smaller average β_{\perp} , although the peak value remains the same. In ICOOL simulations this enables cooling to smaller emittances (~20%), but the acceptance of the channel is slightly reduced.

We have made a first attempt at incorporating the quad channel into the study 2A channel. We replace the solenoidal cooling channel with the FODO cooling channel. A 4-quad match from constant $\beta_{\perp} = 0.8\text{m}$ into the quad channel was generated using DIMAD. (total length of 1.935m) This matches the linear lattice, but does not fully match the transition from solenoidal optics (with amplitude-energy correlations) into separated function FODO optics. The buncher and phase-rotator of Study 2A (with solenoidal B=1.75T optics) was used and the match into the FODO cooler inserted at the end of the phase rotator.

Initial simulations showed that beam was still somewhat mismatched, with the beam somewhat blown up in size after transfer into the FODO cooling channel. The beam then cools in a manner similar to the Study 2A solenoid channel (with LiH absorbers.) The transverse emittance was reduced from 0.0163m to 0.0065m over 100m (compared to 0.0060 for the solenoid case), and the muons within the ECALC9[7] acceptance increased from 0.103 μp to 0.21 over the channel (compared to the increase from 0.126 μp to 0.248 with solenoids). The performance seems a bit inferior to the Study 2A case, with ~10% less beam within the baseline ECALC9 acceptance than the Study 2A case. However the ECALC9 criteria are adapted to cylindrically symmetric geometries with solenoidal focusing; a more appropriate adaptation to FODO optics may modify this assessment.

We therefore rewrote the ECALC9 program as ECALC9xy[8], where x and y emittances and betatron functions are separately calculated. The acceptance criteria are then rewritten as $A_x + A_y < \epsilon_{\text{ref}}$, where A_x and A_y are the separate x and y amplitudes, and ϵ_{ref}

is the reference amplitude. ECALC9 uses the criteria $2A_R < \epsilon_{\text{ref}}$, where A_R is the radially symmetric amplitude. The two criteria obtain roughly the same acceptances for the solenoidal channel, but produces more favorable acceptances in the quad channel evaluations. The muons within the ECALC9_{xy} acceptance increased from 0.108 μ/p to 0.264 over the quad channel (compared to an increase from 0.121 μ/p to 0.258 with solenoids). Under this evaluation, the quad channel is as effective in cooling and beam acceptance as the reference solenoid channel.

The ICOOL simulations indicate the match from the rotator into the FODO cooler is not fully optimized, with some beam blowup occurring at transition. Future optimization may improve the match and reduce losses from mismatch. This study does demonstrate that a FODO channel could be used in initial cooling of muons for a Neutrino Factory, particularly if H_2 absorbers can be used. The FODO channel has some potential advantages in this cooling region over the solenoidal systems, particularly in cost and rf field compatibility.

Use in Buncher and ϕ -E Rotation

We can also consider using quadrupole focusing in the phase energy and rotation section of the Neutrino Factory. In an initial attempt we transferred the solenoid to quad matching section to the front of the bunching section of the Study 2a example and continued quadrupole focusing through the entire bunching, phase-energy rotation, and cooling sections. The results were disappointing. About half of the muons were lost in the first few meters, and that loss of a factor of ~ 2 remained through the system. Most of these were lower-energy muons that apparently were outside the transverse acceptance. Other simulations indicate that the quad channel does not accept lower-momentum muons.

A preliminary conclusion is that the quad channel does not have a large enough momentum acceptance for this section, even though it has an adequate momentum acceptance for the cooling section. This is not unexpected. The momentum acceptance requirements for the cooling channel are $\sim \pm 25\%$; the requirements for the buncher/rotator are closer to $\pm 100\%$.

Further studies should more accurately determine these requirements and whether a quad channel can be redesigned to match these. Further studies could also reoptimize the matching and cooling section parameters for better performance.

References

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Figure 1 sketch of 1.5m FODO transport cell (0.75m half-cell), filled with H_2 gas. Components include F and D quads and a 0.5m region for rf cavities. (The 0.5m length was split into 2 cavities in the simulations.) (Graph shows a radial cross section with $r=0$ at the bottom.)

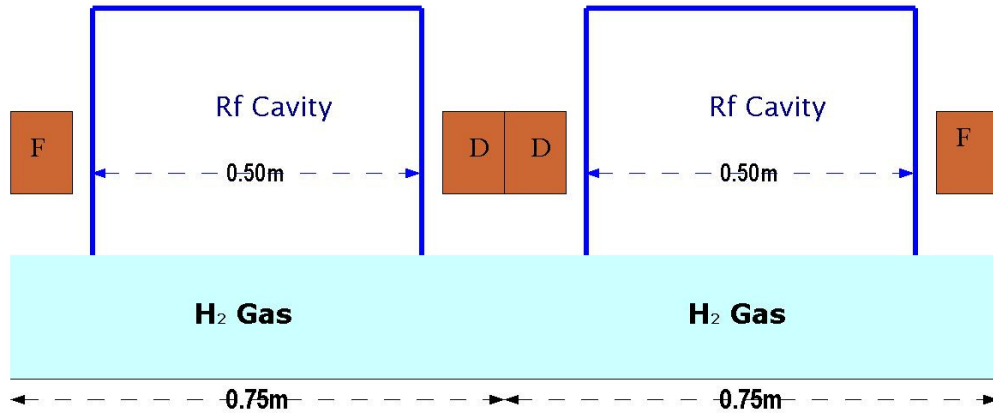


Figure 2. simulation of transverse emittance cooling in a FODO channel with LiH or H_2 absorbers. Vertical scale is distance along the cooling channel (in m) and horizontal scale is rms transverse emittance (normalized) in m.

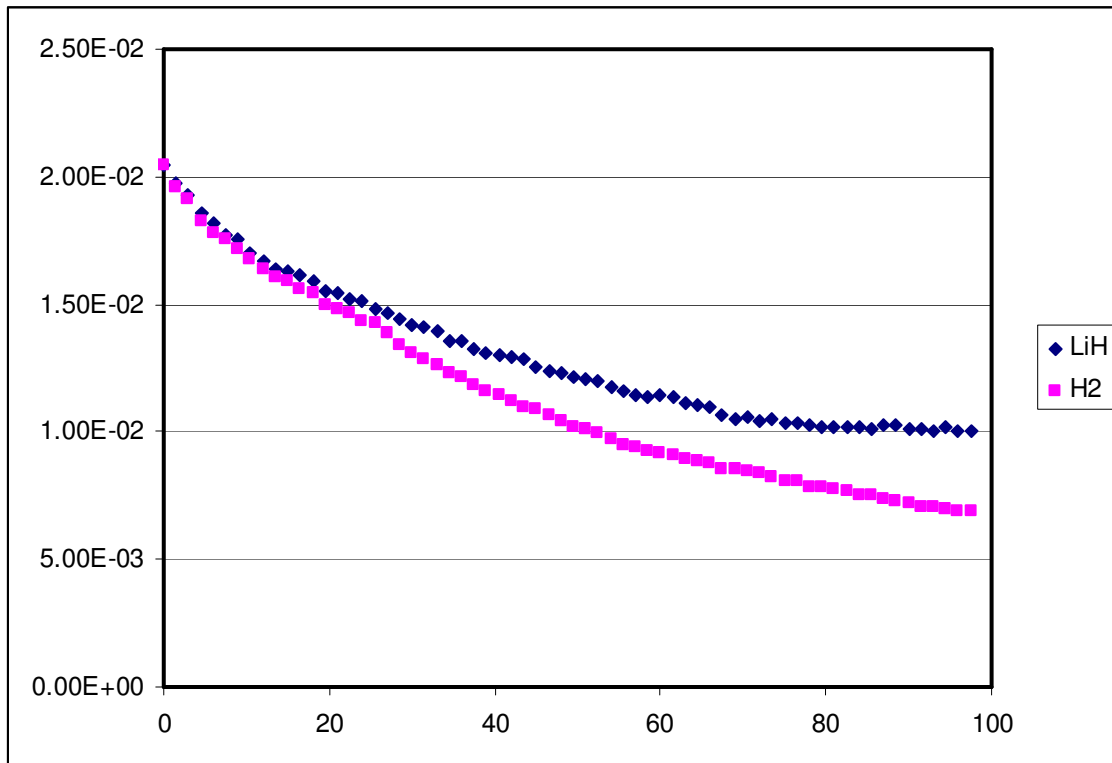


Figure 3. Muon acceptance for criteria under ECALC9xy. Simulation had 5000 initial pions and results are normalized to pions from 5000 initial 24 GeV protons.

n0-Black line – all surviving muons.

n2-Blue line - muons within Study 2A acceptance ($A_x + A_y < 0.03m-R$).

n1-Pink line- muons within Study 2 acceptance ($A_x + A_y < 0.015m-R$).

